Hypothesis Testing about a Population Proportion

1. State the null (H₀) and alternative (Hₐ) hypotheses in plain English

2. State the null and alternative hypotheses using the correct statistical measure (the value of “a” is the hypothesized proportion given in the problem)
   - There are three possibilities:
     - Upper-tailed test – Testing the claim of “greater than”
       \[ H₀: p ≤ a \]
       \[ Hₐ: p > a \]
     - Lower-tailed test – Testing the claim of “less than”
       \[ H₀: p ≥ a \]
       \[ Hₐ: p < a \]
     - Two-tailed test – Testing the claim of “not equal to”
       \[ H₀: p = a \]
       \[ Hₐ: p ≠ a \]

3. Specify the α-level of the test
   - The level of the test determines how rare an event must be in order to reject the null hypothesis
   - This is typically given to you in the statement of the problem
   - Typical values of α are .10, .05, and .01

4. Determine your test statistic:
   - If np ≥ 5 and n(1-p) ≥ 5, use a z-test statistic

5. Determine the critical value of the test statistic and the rejection region
   - This depends on the type of test you are doing (upper, lower or two tailed), the α-level of the test, and the distribution of the test statistic
   - FOR EXAMPLE: α=.05 with a z-test statistic (normal distribution)

   - Upper Tailed Test - Reject the null hypothesis if the sample test statistic is greater than 1.645

   - Lower Tailed Test - Reject the null hypothesis if the sample test statistic is less than -1.645

   - Two Tailed Test - Reject the null hypothesis if the sample test statistic is greater than 1.96 or less than -1.96

6. Compute sample test statistic
   - \( \hat{p} \) -sample proportion
   - \( p \) -hypothesized proportion (“a” from step number 2)
   - \( n \) - sample size
   \[
   z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}
   \]

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7. Make a decision based on your sample test statistic
   • If the value you calculated in step 6 is in the rejection region, reject the null hypothesis in favor of the alternative
   • If the value you calculated in step 6 is NOT in the rejection region, do not reject the null hypothesis
8. State the conclusion in terms of the original question

Example:
Ships arriving in U.S. ports are inspected by Customs officials for contaminated cargo. Assume, for a certain port, 20% of the ships arriving in the previous year contained cargo that was contaminated. A random selection of 50 ships in the current year included 5 that had contaminated cargo. Does the data suggest that the proportion of ships arriving in the port with contaminated cargoes has decreased in the current year? Use \( \alpha = .01 \).

1. **The null hypothesis** – The proportion of ships arriving into the port this year with contaminated cargo is at least \( .20 \)
The alternative hypothesis – The proportion of ships arriving into the port this year with contaminated cargo is less than \( .20 \)
\( H_o: p \geq .2 \)
\( H_a: p < .2 \)
2. \( \alpha = .01 \) (as stated in the problem)
3. Since 50(.2) = 10 and 50(.8) = 40 which are both bigger than 5 we can use a z-test statistic
4. Since 50(.2) = 10 and 50(.8) = 40 which are both bigger than 5 we can use a z-test statistic
5. Since we are conducting a lower-tailed test our rejection region will have a z-critical value of -2.33
6. Our test statistic is:
   \[
   z = \frac{5 - .2}{\sqrt{\frac{.2(1-.2)}{50}}} = \frac{.1}{.0566} = -1.767
   \]
7. Since -2.33 < -1.767 we fail to reject \( H_o \)
8. There is not overwhelming evidence at the \( \alpha = .01 \) level that the proportion of ships arriving to the port this year with contaminated cargo has decreased since last year.